

components are obtained by correlating  $Q_k(nT)$  with  $X_k(nT)$  and with  $Y_k(nT)$ .

If alternate switched transmission is used for measuring the polarimetric variables and reception of the copolar  $H_k(2nT)$ ,  $V_k(2nT+T)$ , and cross-polar components  $Q_k(2nT)$ ,  $Q_k(2nT+T)$  is made then all these signals are whitened in range first and then the usual processing in time (as set forth in Doviak and Zrnic's book entitled "Doppler Radar and Weather Observations", 1993) is made. This entails either averaging the spectrum estimates or various autocovariance estimates in range from which the spectral moments are obtained. For example, to obtain the linear depolarization ratio (LDR) one first whitens the cross polar signal  $Q_k(2nT)$  in range; then the average in range and time produces the power estimate  $P_w$ . By definition  $LDR=P_w/P_h$ , where  $P_h$  is now obtained from the whitened horizontally polarized signals  $X_k(2nT)$ . The multipliers in the argument,  $2n$  and  $2n+1$ , distinguish between the two interlaced sequences of transmitted orthogonal polarization states (i. e., vertical and horizontal).

To construct the whitening matrix  $W$  the following procedures are used. In general the orthogonalization is not unique and many well-known methods could be applied to generate different whitened sequences. The present invention pertains to the specific applications of a whitening scheme on pulsed active remote sensors, such as weather radars, lidars, sodars, blood flow meters, etc., when incorporated into the receivers of these sensors the scheme produces superior estimates of the spectrum, its moments, and polarimetric variables if applicable. Two prominent methods to generate whitened sequences are the eigenvalue decomposition (see the C. W. Therrien 1992 reference publication entitled "Discrete Random Signal And Statistical Signal Processing" published by Prentice Hall, Englewood cliffs, N.J., on page 727, sections 2.6 and 2.7) and the triangular or Gram-Schmidt orthogonalization decomposition as described, for example, in the 1984 reference publication of A. Papoulis, entitled "Probability, Random Variable, and Stochastic Processes, second Edition, McGraw-Hill, Singapore, page 576, section 13.1.

In the eigenvalue decomposition method first the eigenvalues  $\lambda_i$  of the correlation matrix  $C$  are computed and  $C$  is represented as

$$C=U^* L U^t, \quad (11)$$

where  $L$  is a diagonal matrix of eigenvalues,  $U$  is the unitary transformation matrix whose columns are eigenvectors of  $C$ , and the superscript  $t$  signifies the transpose. Then to obtain  $W$  a diagonal matrix  $D$  with elements on the diagonal equal to  $\lambda_i^{-1/2}$  is constructed and

$$W=H^{-1}=D U^t. \quad (12)$$

Triangular or Cholesky decomposition is identical to the Gram-Schmidt orthogonalization (see above publication by Papoulis, 1984). The matrix  $H$  is a lower triangular matrix. Hence the whitening matrix (equation (5)) is also lower triangular. A possible advantage of triangular  $H$  matrices is that whitening can proceed in a pipeline manner, that is computations can start as soon as the first sample is taken and progress through subsequent samples. Non-triangular  $H$  matrices require presence of all data before computations can start.

Simulation of the process starting with the slab signal  $s_i$ , their superposition as in equation (2) and subsequent whitening in range has been made. A pair of such complex signals is simulated with appropriate powers and cross

correlations to represent orthogonal linear polarized echoes. A Doppler spectrum with Gaussian shape is imposed. The standard deviations and mean values of estimates are in FIGS. 4(a)-(c) and 5(a)-(c). FIGS. 4(a)-(c) are standard deviations of spectral moment estimates. In FIG. 4(a) a standard deviation of power, the mean Doppler velocity is shown in FIG. 4(b), and the Doppler spectrum width is shown in the FIG. 4(c) graph. These graphs were obtained by simulating correlated range samples and applying both traditional (thin lines) and proposed (thick lines) processing.  $M$  is the number of time samples (separated by  $T$  seconds) which are used to compute the Doppler spectrum and its moments.  $L$  is the over sampling factor, i.e., the number of range samples that are used to reduce the standard error of estimates. The simulation results were obtained from 1000 realizations. For clarity, lines connect the simulation results (circles at SNR increments of 3 dB). In FIG. 5(a)-(c) are graphs of the mean values of spectral moment estimates. In FIG. 5(a), the top graph is the mean of power, the mean Doppler velocity is shown in FIG. 5(b), and the Doppler spectrum width is shown in the bottom graph, FIG. 5(c). These graphs were obtained by simulating correlated range samples and applying both traditional (thin lines), and the proposed (thick lines) new processing. The imposed spectrum width is  $4 \text{ m s}^{-1}$ , the unambiguous velocity is  $32 \text{ m s}^{-1}$ , the over sampling is by a factor of 10, and the dwell time is 32T s. Comparison of results obtained with standard and proposed processing as a function of SNR demonstrates that for this particular set of parameters (at large SNR) the reduction in standard errors is more than two times (FIGS. 4(a)-(c)).

The correlation of samples in range is influenced by both the receiver filter and the transmitted pulse shape. For a filter with an impulse response  $h_m$  the composite (equivalent) correlation becomes (see publication of Papoulis, 1984)

$$c_m^t = c_m^* h_{-m}^* h_m, \quad (13)$$

where the correlation due to transmitted pulse (which need not be rectangular) is  $c_m$ , and  $m$  is lag in range. This equation can be rewritten in an alternate form as

$$c_m^e = \sum_{k=0}^{L-1} p_k^e p_{k-m}^e, \quad (14)$$

where the equivalent "pulse" weighting function is  $p_k^e = p_k^* h_k$ . Thus  $p_k^e$  is a convolution of the pulse envelope with the receiver impulse response. A simple way to obtain this is to attenuate the transmitted pulse, inject it into the receiver and over sample the output. Substituting the equivalent pulse  $p_k^e$  into equation (3) produces  $c_m$ ; this is done once for a fixed pulse shape and receiver bandwidth. Whitening of the range samples can now be accomplished following the previously described procedure.

Alternate whitening procedures are also possible. An alternate way to produce spectral moment and polarimetric variable estimates using a whitening transformation is now discussed. As mention above, variables of interest are obtained from time series (I, Q) data by an intermediate step involving either correlation or spectral methods. It is possible then to move the whitening transformation one step further in the processing chain.

The modified procedure starts by computing spectral coefficients along sample-time for each set of over sampled data. After computing spectral coefficients, the spectrum peak can be found and only a few spectral components around this peak are needed to determine reflectivity, Dop-